Study of Spin and Charge Fluctuations in the U-t-t′ Model

Charles Buhler and Adriana Moreo
Department of Physics, National High Magnetic Field Lab and MARTECH, Florida State University, Tallahassee, FL 32306, USA

(February 1, 2008)

As additional neutron scattering experiments are performed on a variety of high temperature superconducting compounds it appears that magnetic incommensuration is a phenomenon common to all of the samples studied. The newest experimental results indicate that incommensurate peaks exist at momentum $\mathbf{q} = \pi(1, 1 \pm \delta)$ and $\pi(1 \pm \delta, 1)$ in $La_{2-x}Sr_xCuO_4$ (LSCO) and $YBa_2Cu_3O_{7-\delta}$ (YBCO). The dependence of δ with hole doping appears to be similar in both materials. In addition, new ARPES data for LSCO as a function of doping show that its Fermi surface is qualitatively similar to the one of YBCO, contrary to what was previously believed. Early theoretical attempts to explain LSCO and YBCO behavior usually relied on one- or three-band Hubbard models or the t-J model with electron hopping beyond nearest-neighbor and with different parameter values for each material. In this paper it is shown that using a one band Hubbard U-t-t′ model with a unique set of parameters, U/t=6 and t′/t=-0.25, good agreement is obtained between computational calculations and neutron scattering and ARPES experiments for LSCO and YBCO. It is also shown that using a more negative t′/t will induce short-range magnetic incommensuration along the diagonal direction in the Brillouin zone, in qualitative disagreement with the experimental results. At the finite temperatures of the present Monte Carlo simulation it is also observed that in this model the tendency to incommensuration appears to be more related to the shape of the two-dimensional Fermi surface and the strength of the interaction, rather than to charge order.

PACS numbers: 71.10.Fd, 71.18.+y, 74.25.Ha, 75.40.Mg

I. INTRODUCTION

Neutron scattering experiments continue providing exciting information about the behavior of spin and charge degrees of freedom in the high T_c cuprates. Studies on $La_{2-x}Sr_xCuO_4$ (LSCO) [1] have shown the existence of incommensurability in the spin channel near the commensurate position (π, π). In this case, the incommensurate peaks were found at $Q_\delta = \pi(1 \pm \delta, 1)$ and $\pi(1, 1 \pm \delta)$. The value of δ increases linearly with doping in the range $0.05 \leq x \leq 0.14$, while for $x \geq 0.14$, δ plateaus at 0.25. [2] The intensity at $Q_\gamma = \pi(1 \pm \delta/2, 1 \pm \delta/2)$ was observed to be 0.18 times that at Q_δ for x=0.14 while the intensity at $Q_\pi = (\pi, \pi)$ was negligible compared with the background. [3] For some time neutron scattering experiments in $YBa_2Cu_3O_{7-\delta}$ (YBCO) were less clear. Rossat-Mignod and coworkers detected magnetic fluctuations only at the commensurate position [4], while Tranquada et.al [5] noticed possible incommensurate fluctuations. Measurements by Dai et al. [6] using a new position-sensitive detector bank indicate that incommensurability in the spin channel indeed is present in YBCO. Although originally incommensurability was detected only along the diagonal direction in the Brillouin zone, the latest results indicate that it occurs at Q_δ as in the case of LSCO, and the dependence of δ with doping appears to be similar in both materials. [7] In addition, $Bi_2Sr_2CaCu_2O_8$ has been recently studied by Mook and Chakoumakos [8] and an incommensurate fluctuation that occurs below T_c was found. This incommensuration was identified with a dynamic charge density wave because its scattering intensity appeared to increase with increasing momentum transfer. Scattering that could be described as magnetic has not been observed yet in this material because the experimental technique used does not allow to reach high enough values of the momentum transfer. However, magnetic excitations should be present if a dynamic strip phase gives origin to the charge peaks. Evidence of incommensurability in the charge channel was also observed in $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$ [9] which led to speculations about a similar behavior in LSCO. This would indicate that the incommensuration in the spin channel may be due to the existence of charge-stripe order rather than to some kind of charge uniform spiral spin state or 2D Fermi surface effects. [10]

While some of these experiments suggest that magnetic incommensurability in the cuprates maybe due to charge-stripe order in which the orientation of the stripes is not material dependent, the theoretical understanding of this phenomenon is less clear. Short-range magnetic incommensurate correlations in the spin channel were detected early on in the Hubbard [11,12] and t-J [13] models. The split of the commensurate peak was observed to be qualitatively similar to the behavior in LSCO and YBCO. However, the dependence of δ with doping did not reproduce the experimental data and, as it will be shown later, the experimentally observed relative intensities of the peaks at different points in the Brillouin zone are not well reproduced either. No particular order was reported in the charge channel in these models through

numerical analysis. [14,15] However, since the proposal of dynamical microphase separation in the CuO_2 planes [16], a reanalysis of these conclusions is needed.

At the same time that the recent neutron scattering experiments were discovering new similarities between the magnetic properties of the different cuprates, also recent angular-resolved photoemission (ARPES) experiments performed on LSCO mapped its Fermi surface (FS) [17] at different values of doping unveiling interesting similarities among the qualitative FS shape of several high T_c materials.

Motivated by the new experimental results the aim of this paper is to revisit the U-t-t$'$ model, exploring numerically the spin and charge channels to determine whether it is possible to obtain agreement with the new data for LSCO and YBCO using a unique set of parameters in the model. It will be concluded that this is indeed possible. The paper is organized as follows: in Section II the $U-t-t'$ Hamiltonian and the notation used are introduced. Results on magnetic and charge correlations, as well as on the shape of the Fermi surface, are presented in Section III while Section IV is devoted to the conclusions.

II. THE MODEL

The U-t-t$'$ one band Hubbard model Hamiltonian is given by

$$H = -t \sum_{<\mathbf{ij}>,\sigma} (c^{\dagger}_{\mathbf{i},\sigma} c_{\mathbf{j},\sigma} + \text{h.c.}) - t' \sum_{<\mathbf{in}>,\sigma} (c^{\dagger}_{\mathbf{i},\sigma} c_{\mathbf{n},\sigma} + \text{h.c.}) +$$

$$U \sum_{\mathbf{i}} (n_{\mathbf{i}\uparrow} - 1/2)(n_{\mathbf{i}\downarrow} - 1/2) + \mu \sum_{\mathbf{i},\sigma} n_{\mathbf{i}\sigma}, \qquad (1)$$

where $c^{\dagger}_{\mathbf{i},\sigma}$ creates an electron at site $\mathbf{i}$ with spin projection σ, $n_{\mathbf{i}\sigma}$ is the number operator, the sum $\langle \mathbf{ij} \rangle$ runs over pairs of nearest neighbor lattice sites, and the sum $\langle \mathbf{in} \rangle$ runs over pairs of lattice sites along the plaquette diagonals. U is the on site Coulombic repulsion, t the nearest neighbor hopping amplitude, t$'$ the diagonal hopping amplitude, and μ the chemical potential. In this work t will be set equal to 1.

The static charge and magnetic structure factors $N(\mathbf{q})$ and $S(\mathbf{q})$ are defined by the relations

$$N(\mathbf{q}) = \sum_{\mathbf{r}} e^{i\mathbf{q}\cdot\mathbf{r}} \langle \delta n_{\mathbf{0}} \delta n_{\mathbf{r}} \rangle, \qquad (2)$$

$$S(\mathbf{q}) = \sum_{\mathbf{r}} e^{i\mathbf{q}\cdot\mathbf{r}} \langle S^z_{\mathbf{0}} S^z_{\mathbf{r}} \rangle, \qquad (3)$$

where $\langle \delta n_{\mathbf{0}} \delta n_{\mathbf{r}} \rangle$ and $\langle S^z_{\mathbf{0}} S^z_{\mathbf{r}} \rangle$ are equal-time density- and spin-correlation functions, $S^z_{\mathbf{r}} = \frac{1}{2} \sum_{\alpha,\beta} c^{\dagger}_{\mathbf{r},\alpha} \sigma^z_{\alpha,\beta} c_{\mathbf{r},\beta}$, and $\delta n_{\mathbf{r}} = \sum_{\sigma} c^{\dagger}_{\mathbf{r},\sigma} c_{\mathbf{r},\sigma} - \langle n \rangle$. Here $\langle n \rangle = 1 - x$ is the average density of electrons. The brackets in Eqs. (2) and (3) refer to thermal averaging in the grand canonical ensemble which will be performed using the standard Quantum Monte Carlo (QMC) determinantal method.

Before presenting our results let us discuss the behavior of the spin and charge correlations in the non-interacting system (U/t=0), and also in the standard Hubbard model with t$'$=0. In the non-interacting system the spin and charge correlations are related through $S(\mathbf{q}) = \frac{1}{4} N(\mathbf{q})$. $N(\mathbf{q})$ increases from zero reaching the value $\langle n \rangle$ at $q = 2k_F$ and remaining constant afterwards. On the other hand, in the Hubbard model it was observed that at low density, i.e., $\langle n \rangle < 0.5$, $S(\mathbf{q})$ peaks at $q = 2k_F$ while $N(\mathbf{q})$ is suppressed at these momenta compared with the non-interacting system and it only peaks at $\mathbf{q} = (\pi, \pi)$. [14] This peak is due to the short range effective repulsion between particles. The behavior at higher densities is very different in the spin channel. At half-filling a sharp peak develops at Q_π and, with a substantially reduced intensity, it moves to Q_δ with doping. While the intensity is maximum at Q_δ as in the experiments, the weight at Q_π is always larger than the one at Q_γ [11] which is qualitatively incorrect compared with recent experiments as it will be shown below.

III. RESULTS

Due to the well known "sign problem" it is very difficult to perform Monte Carlo numerical studies at small hole doping, low temperatures and values of $U/t > 4$. This problem is exacerbated as the absolute value of t$'$ increases. For this reason the numerical efforts will be concentrated here on the study of the fixed density $\langle n \rangle = 0.7$, (i.e., x=0.3) since for this $\langle n \rangle$ a good degree of control of the numerical results can be achieved. In addition, new experiments have been performed in LSCO at precisely x=0.3 providing information about incommensuration [2] and the shape of the FS [17]. Here results for U/t=6 on 8×8 lattices will be presented. Due to the sign problem at this relatively large value of U/t, and using a finite diagonal hopping t$'$, the temperature had to be fixed at T=0.25t. It is to be expected that the intensity of the observed features will increase at lower temperatures and the results presented here are, thus, lower bounds to the actual values.

Note that, as part of our study, runs for other values of U/t were performed and qualitative differences with the results for U/t=6 were not observed. In particular, for $t'/t = 0$ we found that for values of U/t as high as 10 the magnetic incommensuration always occurs at Q_δ rather than along the diagonal as predicted by mean-field calculations in the strong coupling regime. [18]

A. Magnetic Incommensurability

As a first step, the static structure factor will be calculated for several values of t′/t and comparisons with the experimental neutron scattering results will be made along several directions in momentum space.

In Fig.1-a the peaks in $S(\mathbf{q})$, indicative of short range spin incommensurate tendencies in the U − t − t′ model, are presented along the $(0,\pi)-(2\pi,\pi)$ direction for values of t′/t ranging from 0 to -0.5. The figure shows that for all values of t′/t analyzed here, the peak in the structure factor occurs at $\mathbf{q}=(3\pi/4,\pi)$ and $(5\pi/4,\pi)$ which correspond to $\delta=0.25$. This is in agreement with the experimental value for x=0.3 in LSCO. [2] However, a spline fit through the available data points (dashed line) suggests that the actual peak at $|t'/t|\geq 0.3$ occurs at a slightly larger value of δ. Actually, when the structure factor is scanned along the diagonal direction as shown in Fig.1-b it is clear that the results with $|t'/t|=0.3$ or larger do not fit the experimental data because the maxima in $S(\mathbf{q})$ at $\mathbf{q}=(1\pm\delta)\pi(1,1)=(3\pi/4,3\pi/4)$ and $(5\pi/4,5\pi/4)$ have intensities which are approximately equal or higher than those at Q_δ [19], as it can be deduced by comparing Fig.1-a with Fig.1-b. This is in disagreement with the experiments that indicate that the intensity at $\mathbf{q}=(1\pm\delta)\pi(1,1)$ should be indistinguishable from the background. [2,3,6,7]

An important result is that at t′=0 $S(\mathbf{q})$ along the diagonal direction, $\mathbf{q_x}=\mathbf{q_y}$, has a maximum at $\mathbf{q}=Q_\pi$ as it can be observed in Fig.1-b (top curve). This behavior is in disagreement with the experimental data for LSCO presented in Fig.3 (closed circles) of Ref. [3], and for YBCO in Fig.1-c of Ref. [6] where a minimum is observed at Q_π along the diagonal. The experimental results were obtained at x=0.15 for optimally doped LSCO and x=0.1 (i.e., $\delta=0.4$) for YBCO showing that the qualitative behavior does not depend strongly on doping. This indicates that the standard Hubbard model (t′ = 0) does not describe the qualitative behavior observed with neutron scattering when $\delta=0.25$. However, working with t′=-0.2 the fit along the diagonal in momentum space (Fig.1-b) indicates that $S(Q_\pi)$ now has become a local minimum, in qualitative agreement with the experiments. Then, the constraints on the relative intensity of the peaks provided by the experiments leave a small window of possible values of t′/t. If $|t'/t|\geq 0.3$ the incommensurate peaks will appear along the diagonal rather than at Q_δ (as discussed in the previous paragraph) and if $|t'/t|\leq 0.2$ the structure factor at Q_π would be a local maximum rather than a minimum along the diagonal direction.

Additional experimental measurements [3] have been reported along the direction $Q_\delta[=\pi(1-\delta,1)]\rightarrow Q_\gamma[=\pi(1-\delta/2,1+\delta/2)]\rightarrow Q_\delta[=\pi(1,1+\delta)]$. Note that for $\delta=0.25$ the point Q_γ is given by $(7\pi/8,9\pi/8)$ which is computationally accessible only on 16×16 or larger lattices if periodic boundary conditions are being used. Since such a large lattice size is beyond the capabilities of the QMC at finite hole density we obtained the values of $S(\mathbf{q})$ at the equivalent Q_γ points located at $7\pi/8(1,1)$ and $9\pi/8(1,1)$ with the help of a spline fit of the available data in Fig.1-b (dashed line).

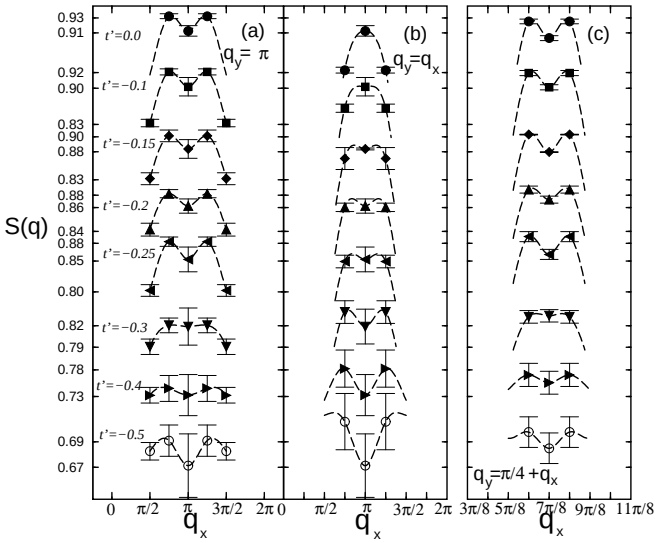

FIG. 1. The static structure factor $S(\mathbf{q})$ for U/t=6, $\langle n\rangle=0.7$ on an 8×8 lattice for T=0.25t and values of t′/t ranging from 0 to -0.5. a) Along the $(\pi,0)-(\pi,2\pi)$ direction; b) along the diagonal direction; c) along the $q_y=\delta\pi+q_x$ direction. The dashed line indicates a spline fit.

In Fig.1-c the structure factor is shown along the line $q_y=\delta\pi+q_x$ which corresponds to the direction along which the experimental neutron scattering data shown by open circles in Fig.3 of Ref. [3] were taken. In the figure it can be seen that the relative intensity of the numerical data at $q_x=\pi(1-\delta/2)=7\pi/8$, (i.e., Q_γ) and $q_x=\pi(1\pm\delta)=0.75\pi$ or 1.25π, (i.e., Q_δ) is a function of t′. Considering, as in the experiments, that the intensity for $\mathbf{q}=\pi(1\pm\delta)(1,1)$ (this point corresponds to $q_x=3\pi/4$ and $5\pi/4$ in Fig.1-b) has to correspond to the background, then the relationship $Q_\gamma/Q_\delta=0.18$ is satisfied for t′/t=-0.25. This is used as a guide because this relationship may be doping dependent but since the window in t′/t is so narrow, at most an error of the order of 0.05 in the estimation of t′/t is being made.

The above analysis shows that a comparison of the present numerical data when the measured incommensurability δ is 0.25 agrees with the data for *LSCO* with the same δ using U/t=6 and t′/t=-0.25. Notice that the position and the relative intensity of the peaks does not change too much with temperature according to the experiment.

B. Fermi Surface

The next issue that will be addressed is whether the ratio $t'/t = -0.25$, fixed by the spin structure factor analysis in the previous subsection, will fit other experimental data such as, for example, the shape of the Fermi surface recently obtained using ARPES for LSCO. [17] The possible shape of the FS will be determined by analyzing the momentum distribution $n(\mathbf{q})$ which is calculated by Fourier transforming the one-electron Green's function,

$$g_{\mathbf{ij}} = -\langle \sum_{\sigma} c_{\mathbf{i},\sigma} c^{\dagger}_{\mathbf{j},\sigma} \rangle, \tag{4}$$

that is evaluated using QMC.

The criteria used here to obtain the most probable locus of the FS from numerical $n(\mathbf{q})$ data are two: (a) find the values of $\mathbf{q}$ where $n(\mathbf{q}) \approx 0.5$ [11] and (b) find the values of $\mathbf{q}$ where $n(\mathbf{q})$ changes the most rapidly [20]. For the case of t'/t=-0.25 we have observed that both methods provide similar results in the regions close to the diagonal direction in the Brillouin zone, but substantial differences were observed close to the $(0, \pi)$ and $(\pi, 0)$ points. While criterion (a) indicated a FS closed around (0,0), criterion (b) indicated a FS closed around (π, π). Since criterion (b) provided similar results for the non-interacting case with $\langle n \rangle = 0.7$ and t'/t=-0.25, a situation where it is known that the FS actually closes around (0,0), it was decided that criterion (a) would be more effective in this context.

The FS obtained with procedure (a) is shown in Fig.2-a. The closed circles indicate where $n(\mathbf{q}) \approx 0.5$ and the continuous line is a 6th order polynomial fit of the points. The obtained FS is very similar to the non-interacting one (denoted by a dashed line in the same figure). It satisfies Luttinger's theorem within error bars and, actually it is in excellent agreement with the experimental data for x=0.3 [17] shown with open squares in Fig.2-a. Thus, using $t'/t = -0.25$ good agreement has been obtained between the numerical results and two independent experiments (neutron scattering and ARPES) performed in overdoped LSCO. Since the obtained Fermi surface resembles closely the non-interacting one, the FS in the U/t=0 limit was calculated for x=0.1 and its shape compared directly with the experimental results at this density. As it can be observed in Fig.2-b the agreement is once again very good. According to Ref. [17] the FS for LSCO at optimal doping is still centered about (π, π) and it is qualitatively similar to the one obtained for x=0.1. This is indeed what happens with the U/t=0 FS for t'/t=-0.25 at x=0.15 which is shown in Fig.2-c (dashed line). At the non-interacting level the change between a FS that closes around (π, π) and around $(0, 0)$ occurs at x=0.22 for the ratio of t'/t used here. In addition, in Fig.2-c the experimental points for optimal doped YBCO obtained several years ago [22] are also shown.

The agreement with our result is only qualitative but it has to be considered that the measurements are very difficult due to surface effects and thus the experimental points have large error bars (not shown). It is important to remark that the similarity found between the interacting and the non-interacting FS for t'/t=-0.25 does not mean that they are identical but the differences will be apparent only when larger lattices at lower temperatures can be studied.

Sometimes the experimental results have been interpreted as indicating that there is a FS only along the diagonal direction and no FS close to $(0, \pi)$ and $(\pi, 0)$. [21] The rate of change of $n(\mathbf{q})$ could support this view but we found out that it also gets very reduced close to $(0, \pi)$ and $(\pi, 0)$ even in the non-interacting case when it is known that there is a continuous FS. Thus, the present results do not allow us to decide one way or the other. We also noticed that the interacting FS for t'/t=-0.3 appears to be qualitatively different from the non-interacting one. In particular at x=0.3 it seems to close around (π, π) while the non-interacting one closes around $(0, 0)$. Thus, if YBCO were to have the same qualitative shape of FS than LSCO in the overdoped regime this is another reason to rule out values of $|t'/t|$ equal or higher than 0.3.

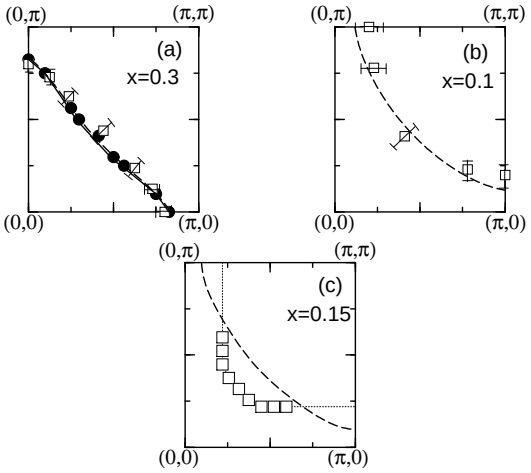

FIG. 2. (a) The numerically calculated Fermi surface for U/t=6, $\langle n \rangle = 0.7$ on an 8×8 lattice for T=0.25t and t'/t=-0.25 (closed circles and solid line); the open squares are experimental results for LSCO at x=0.3 from Ref. [17]; the dashed line is the non-interacting, U/t=0, FS for t'/t=-0.25 and $\langle n \rangle = 0.7$; (b) non-interacting (U/t=0) FS for t'/t=-0.25 and density 0.9 (dashed line) and experimental data for LSCO with x=0.1 from Ref. [17]; (c) non-interacting FS for t'/t=-0.25 and density 0.85 (dashed line) together with experimental data for YBCO with $x \approx 0.15$ from Ref. [22].

C. Charge Correlations

The next issue that will be addressed is the origin of the incommensurate magnetic fluctuations in the present model. In Fig.3 the charge structure factor $N(\mathbf{q})$ along the $(0,0) \rightarrow (\pi,\pi) \rightarrow (\pi,0) \rightarrow (0,0)$ directions for t$'$/t ranging from 0 (top) to -0.5 (bottom) is shown. In all cases there is a broad maximum at Q_π which is due, as in the low electron density limit of the Hubbard model, to the short range effective repulsion between particles.

If the incommensurate magnetic fluctuations were due to dynamical charge fluctuations, peaks at $\mathbf{q} = \pi(0, 2\delta)$ and $\pi(2\delta, 0)$ should be observed in $N(\mathbf{q})$ according to previous theoretical studies [9,16]. In the present case, since $\delta = 0.25$, the peaks would be expected at $(0, \pi/2)$ and $(\pi/2, 0)$. This momentum is indicated with an arrow in Fig.3 and it is clear from the figure that no indications of incommensurate charge order is observed.

Another possible origin of the magnetic incommensuration in 2D could be simple FS effects. [10,23,24] There are some momenta that map points (or regions) of the FS into other points (or regions) also on the FS. These are called nesting or pseudonesting vectors and they correspond to values of momentum where maxima occur in the imaginary part of the magnetic susceptibility in the 2D non-interacting system. We performed a numerical calculation of the pseudonesting vector for the interacting FS. We constructed a histogram in order to identify the value of the momentum that mapped most points of the FS into other points also belonging to it. For t$'$/t=-0.25 at x=0.3 a maximum in the histogram was obtained at $\mathbf{q} = (\pi, 0.67\pi)$ which is in very good agreement with the analytical value for the corresponding non-interacting FS, namely $\mathbf{q} = (\pi, 0.71\pi)$ [23]. Thus, the maximum in $S(\mathbf{q})$ at $\mathbf{q} = (\pi, 0.75\pi)$ could be explained by FS effects in this case but it may be due to the coarse grid that necessarily had to be used in our computational studies. In the non-interacting case it is expected that the maximum remains at Q_π until x reaches 0.22 (see Sect.III.B). Though this behavior seems to be in agreement with previous results for the interacting case [20] and in disagreement with the experimental data, it is possible that the effect of the interaction at smaller dopings will be observed at lower temperatures than the ones that can presently be reached.

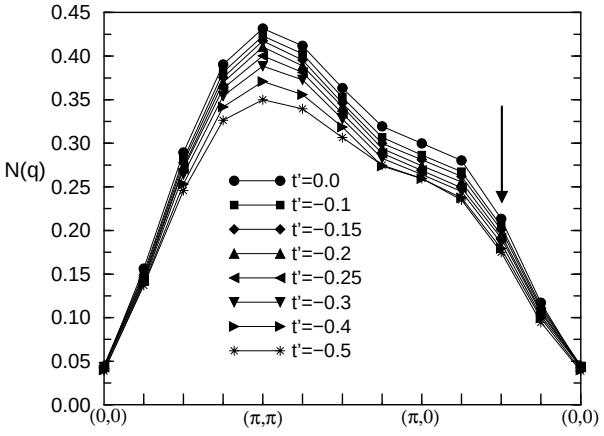

FIG. 3. Static charge structure factor for U/t=6, $\langle n \rangle = 0.7$ on an 8×8 lattice for T=0.25t and values of t$'$/t ranging from 0 (top) to -0.5 (bottom) along the directions $(0,0)-(\pi,\pi)-(\pi,0)-(0,0)$. The arrow indicates the value of the momentum where a maximum indicating incommensurate short-range order would be expected.

IV. CONCLUSIONS

In summary, motivated by recent neutron scattering and ARPES experiments, we have studied the U-t-t$'$ model numerically on 8×8 lattices at temperature T=0.25t and with coupling U/t=6. Fixing the density at $\langle n \rangle = 0.7$ agreement between the position and the relative intensity of the incommensurate peaks obtained numerically and experimentally for LSCO and YBCO is observed for t$'$/t=-0.25. Larger values of |t$'$/t| are ruled out because in this case the structure factor has a maximum along the diagonal direction rather than at Q_δ, while with less negative values of t$'$/t a relative maximum is observed at Q_π along the diagonal direction, again in disagreement with experimental results. Thus, the Hubbard or t-J models without nearest-neighbor electron hopping do not reproduce appropriately this experimental behavior in the cuprates.

The addition of a diagonal hopping t$'$/t=-0.25 also provides good agreement with experimental measurements of the FS for LSCO and YBCO at different densities. A FS that closes around (π,π) in the underdoped and optimal doped regimes, and around $(0,0)$ in the overdoped case is observed.

The incommensurate magnetic peaks do not seem related to incommensurate charge fluctuations. In the overdoped regime FS effects enhanced by the electronic interactions may be responsible for the observed results. The sign problem prevented the exploration of this issue in the optimal doped and underdoped regimes.

The present numerical analysis provides good agreement between a theoretical model and two unrelated experiments in the overdoped regime of LSCO. It also shows that the consideration of a diagonal hopping in models

for the cuprates is crucial in order to reproduce experimental data. Note that this result is in excellent agreement with ARPES calculations that have focussed on the insulating compound $Sr_2CuO_2Cl_2$. [25] For the cases analyzed here, the same ratio t'/t can reproduce results for both LSCO and YBCO. Although the behavior of the incommensurate magnetic peaks in the cuprates appear to be similar, there is still no experimental information about relative intensities of the peaks at different points in the Brillouin zone carried out at the same density. These measurements could indicate possible material dependent properties that could depend on longer range electron hopping terms. [25]

V. ACKNOWLEDGEMENTS

We would like to acknowledge useful discussions with P. Dai, G. Aeppli, T. Mason, H. Mook, Q. Si, J. Tranquada, M. Randeria, M. Norman and E. Dagotto. A.M. is supported by NSF under grant DMR-95-20776. Additional support is provided by the National High Magnetic Field Lab and MARTECH.

[1] S-W. Cheong et al., Phys. Rev. Lett. **67**, 1791 (1991); T.E. Mason et al., ibid. **68**, 1414 (1992).
[2] K. Yamada et al., Phys. Rev. **B57**, 6165 (1998).
[3] T.E. Mason, et al., Phys. Rev. Lett. **71**, 919 (1993).
[4] J. Rossat-Mignod et al., Physica (Amsterdam) **185C**, 86 (1991).
[5] J.M. Tranquada et al., Phys. Rev. **B42**, 5561 (1992); B.J. Sternlieb et al., ibid **50**, 12915 (1994).
[6] P. Dai, H.A. Mook and F. Dogan, Phys. Rev. Lett. **80**, 1738 (1998).
[7] H.A. Mook, talk presented at the March Meeting 1998, Los Angeles; H.A. Mook, P. Dai, S.M. Hayden, G. Aeppli, T.G. Perring and F. Dogan, preprint.
[8] H.A. Mook and B.C. Chakoumakos, J. of. Superconductivity **10** 389 (1997).
[9] J.M. Tranquada et al., Phys. Rev. Lett. **78**, 338 (1997).
[10] P.B. Littlewood, J. Zaanen, G. Aeppli and H. Monien, Phys. Rev. **B48**, 487 (1993).
[11] A. Moreo et al., Phys. Rev. **B41** 2313 (1990).
[12] M. Imada and Y. Hatsugai, J. Phys. Soc. Jpn. **58**, 3752 (1989).
[13] A. Moreo et al., Phys. Rev. **B42** 6283 (1990).
[14] Y.C. Chen et al., Phys. Rev. **B50**, 655 (1994).
[15] W.O. Putikka et al., Phys. Rev. Lett. **73**, 170 (1994).
[16] V.J. Emery and S.A. Kivelson, Physica **C209**, 597 (1993); ibid **C235** 189,1994.
[17] A. Ino, C. Kim, T. Mizokawa, Z.X. Shen, A. Fujimori, M. Takaba, K. Tamasaku, H. Eisaki and S. Uchida, preprint.
[18] A. Chubukov and K. Musaelian, Phys. Rev. **B51**, 12605 (1995).
[19] Calculations using the Composite Operator Method show that this behavior occurs for t'/t=-0.1. See A. Avella, F. Mancini and D. Villani, preprint cond-mat/9707266.
[20] D. Duffy and A. Moreo, Phys. Rev. **B52**, 15607 (1995).
[21] X.G. Wen and P.A. Lee, Phys. Rev. Lett. 80, 2193 (1998); V.B. Geshkenbeim, L.B. Ioffe and A.I. Larkin, Phys. Rev. **B55**, 3173 (1997); J.R. Engelbrecht et al., cond-mat/9705166; W.O. Putikka, M.U. Luchini and R.R.P. Singh, cond-mat/9803140.
[22] J.C. Campuzano et al., Phys. Rev. Lett. 64, 2308 (1990).
[23] P. Benard, L. Chen and A.-M. Tremblay, Phys. Rev. **B47**, 15217 (1993).
[24] Q. Si, Y. Zha, K. Levin and J.P. Lu, Phys. Rev. **B47**, 9055 (1993).
[25] A. Nazarenko, K. Vos, S. Haas, E. Dagotto and R. Gooding, Phys. Rev. **B51**, 8676 (1995); O. Starykh, O. de Alcantara Bonfim, and G. Reiter, Phys. Rev. **B 52**, 12534 (1995); B. Kyung and R. Ferrell, *ibid* **54**, 10125 (1996); T. Xiang and J. M. Wheatley, *ibid* **54**, R12653 (1996); T. K. Lee and C. T. Shih, *ibid* **55**, 5983 (1997); P. W. Leung, B. O. Wells, and R. J. Gooding, preprint; R. Eder et al., preprint; C. Kim, P. J. White, Z.-X. Shen, T. Tohyama, Y. Shibata, S. Maekawa, B. O. Wells, Y. J. Kim, R. J. Birgeneau, and M. A. Kastner, preprint.